

# AdS/QCD models describing a finite number of excited mesons with Regge spectrum

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## Abstract

The typical AdS/QCD models deal with the large- $N_c$  limit of QCD, as a consequence the meson spectrum consists of the infinite number of states that is far from the real situation. Basing on introduction of anharmonic corrections to the holographic potential, the corrections whose existence has been recently advocated, we construct a class of bottom-up holographic models describing arbitrary finite number of states in the sector of light mesons. Within the proposed approach, the spectrum of masses square has the following properties: It is linear,  $m_n^2 \sim n$ , at not very large  $n$ , nonlinear at larger  $n$ , with the nonlinear corrections being subleading in  $1/N_c$ , has a limiting mass, and the number of states is proportional to  $N_c$ . The considered holographic models reflect thereby the merging of resonances into continuum and the breaking of gluon string at sufficiently large quark-antiquark separation that causes the linear Regge trajectories to bend down. We show that these models provide a correct description for the spectrum of excited  $\rho$ -mesons.

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# 1 Introduction

Recently the holographic models of QCD have been successfully applied to the description of non-perturbative physics of the strong interactions [1–4]. For this reason it is interesting to further investigate to what extent such a phenomenological approach (often called AdS/QCD) is able to describe the low energy QCD irrespective of whether it is related to the original AdS/CFT correspondence [5] or not.

Maldacena’s hypothesis [5] provided a promising way to link the fundamental string theories in the low energy approximation to the strongly coupled four-dimensional conformal field theories living on the boundary of AdS space [6]. As the AdS/CFT method permits to obtain a theoretical control over the strongly coupled gauge theories, the next natural step is to extend the idea of holographic correspondence to the physical gauge theories, such as QCD. This step was realized, albeit speculatively, through the AdS/QCD models (see, *e.g.*, [7] for a brief review). The latter approach, however, deals typically with the planar limit of QCD,  $N_c \rightarrow \infty$ , *i.e.* it is rather restricted from the very beginning (see discussions in [8] on that point). As a consequence, the meson spectrum in the AdS/QCD models consists of an infinite number of infinitely narrow states in a sharp contradiction with what we observe experimentally: There are a few of resonances in each channel, they have a finite width and the discrete spectrum gradually merges into perturbative continuum. In the program of relating the strings to the real hadron physics, the next step is therefore called for — the development of holographic models describing a finite number of hadrons merging into continuum. In the present Letter, we will construct a class of bottom-up holographic models possessing this property.

The paper is organized as follows. In Sect. 2, we formulate the problem and specify the subject matter of this work. Sect. 3 is devoted to the construction of a general design for the holographic models describing a finite number of mesons in the simplest case. Using the modern experimental data we show in Sect. 4 that the found form of spectrum seems to be supported experimentally. A simple particular realization of the proposed idea is considered in Sect. 5. The results obtained, their relation to other works, and the possible consequences are discussed in Sect. 6. Sect. 7 concludes our analysis.

## 2 Formulation of the problem

The problem of primary importance in the non-perturbative QCD is the description of spectrum of light hadrons. In this work, we will be interested only in the spectroscopy of light resonances, namely we will consider the light mesons because their spectrum is well defined in the large- $N_c$  limit. Experimentally, the most investigated sector is that of the  $\rho$ -mesons [9], for this reason we will restrict ourselves to the  $\rho$ -mesons only since only in this case there is enough experimental data to make more or less reliable fits.

The QCD coupling constant  $\alpha_s$  is running, this fact makes impossible to follow the principles of AdS/QCD correspondence exactly, nevertheless, one can attempt to follow these principles as close as possible. According to the averaged fit [9], the value of  $\alpha_s$  at the scale of the  $\rho$ -meson mass is  $\alpha_s(0.776 \text{ GeV}) \approx 0.71$ , at the scale of the first radial excitation of the  $\rho$ -meson  $\alpha_s(1.465 \text{ GeV}) \approx 0.35$ , at the scale of the highest observed radial excitations  $\alpha_s(2.300 \text{ GeV}) \approx 0.28$ . It is seen that in the energy region where the most resonances reside the change of  $\alpha_s$  is moderate (about 20%), on the other hand, the theory is strongly coupled (there are resonances), hence, a weakly coupled holographic model dual to QCD in that region has chances to exist in some approximate sense. Below that region QCD is strong but  $\alpha_s$  changes rapidly (see, however, the recent indications on the existence of infrared fixed point [10] suggesting  $\frac{\alpha_s}{\pi} \lesssim 1$ ), above the resonance region the two-flavor QCD is approximately scale invariant but is in the weakly coupled regime, in both cases the principles of AdS/QCD correspondence are badly violated. Thus, we are going to describe holographically the resonance region that is restricted by the infrared,  $\Lambda_{\text{IR}}$ , and the ultraviolet,  $\Lambda_{\text{UV}}$ , cutoffs. The introduction of the UV-cutoff separating the resonance region from the onset of the perturbative continuum is crucial for our further analysis and distinguishes our models from other AdS/QCD models existing in the literature. The IR-cutoff is fixed by hand in the hard-wall models [1, 2, 11] in order to model the confinement, in our case an effective IR-cutoff will be introduced in a different way.

Considering QCD with  $SU(N_c)$  gauge group, where  $N_c$  is finite and arbitrary, we must answer the following question: How many resonances should we take into account? The question is not trivial because, generally speaking, the cutoff  $\Lambda_{\text{UV}}$  depends on  $N_c$ . However, there is a simple argument [12] that if the spectrum of masses square is linear,  $m_n^2 \sim \mu^2 n$  (as expected from the phenomenology [13] and the semiclassical string models for mesons [14]) the number of resonances should be proportional to  $N_c$ . Indeed, in the Breit-Wigner parametrization of resonances, the width is  $\Gamma_n m_n$  and if this quantity becomes equal to the distance between masses square of the neighboring res-

onances,  $\Gamma_n m_n \sim \mu^2$ , we cannot speak any more about resonances as they merge into continuum. On the other hand, according to the semiclassical string (flux-tube) models [14]  $\Gamma_n \sim m_n/N_c$ , hence,  $\Gamma_n m_n \sim m_n^2/N_c \sim \mu^2 n/N_c$  and we obtain  $n_{\max} \sim N_c$ . This expectation will be an important input in what follows.

### 3 Construction of model

In order to advance in holographic description of real hadron physics one usually extends the existing successful holographic models. There are two complementary ways for such extensions, either by introducing new operators/fields following the AdS/CFT principles or by modification of the background metric. The former seems to be more consistent but at the same time more difficult and somewhat ambiguous at the present stage. The latter seems rather *ad hoc* but simpler, for this reason we will exploit this geometric way hoping that subsequently the results obtained will be justified from a more fundamental point of view with the help of introduction of appropriate operators/fields. We note in advance that the first step in justifying our approach has been already done in [15].

The simplest 5d action describing the spectrum of vector mesons is [2]

$$S = \int d^5x \sqrt{g} \left( -\frac{1}{4g_5^2} F_{MN} F^{MN} \right), \quad (1)$$

where  $g = |\det g_{MN}|$  and  $F_{MN} = \partial_M V_N - \partial_N V_M$ . Generally speaking, from the 5d side we must have a Yang-Mills theory with  $SU(N_f)$  gauge group [2] but for calculation of the mass spectrum one retains the quadratic in fields part only so that the Abelian part of  $F_{MN}$  is enough for our purposes. The IR boundary condition is that the action is finite at  $z = \infty$ . The metric is parametrized as

$$g_{MN} dx^M dx^N = e^{2A(z)} (dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu), \quad (2)$$

with  $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ . The equation of motion for action (1) possesses a solution for the string modes  $V_M(x, z)$  which is supposed to be dual to physical states of the gauge theory. Fixing the gauge  $V_z = 0$ , the corresponding equation reads [3]

$$\partial_z (e^A \partial_z v_n) + q^2 e^A v_n = 0, \quad (3)$$

where  $v_n$  must be normalizable solutions for the 4d-transverse components  $V_\mu^T$  which exist only for discrete values of 4d-momentum  $q^2 = m_n^2$ . Performing the substitution

$$v_n = e^{-A/2} \psi_n, \quad (4)$$

Eq. (3) takes the form of a Schrödinger equation,

$$-\psi_n'' + U(z)\psi = m_n^2\psi_n, \quad (5)$$

with the potential

$$U(z) = \frac{1}{4}(A')^2 + \frac{1}{2}A''. \quad (6)$$

Thus the spectrum of the 4d gauge theory and the metric of the dual 5d theory are related (up to boundary conditions) by the form of potential (6). The fifth coordinate  $z$  is known to correspond to the energy scale  $Q$ :  $Q \sim 1/z$ . Let us divide the potential  $U(z)$  into the UV and IR parts,

$$U(z) = U_{\text{UV}}(z) + U_{\text{IR}}(z), \quad (7)$$

where  $U(z) \xrightarrow{z \rightarrow 0} U_{\text{UV}}(z)$  and  $U(z) \xrightarrow{z \rightarrow \infty} U_{\text{IR}}(z)$ .

As follows from the AdS/CFT correspondence, the vector wave functions must have the UV asymptotics  $\psi(z) \xrightarrow{z \rightarrow 0} z^2$  [11], this dictates  $U_{\text{UV}}(z) \xrightarrow{z \rightarrow 0} z^{-2}$ . The usual choice is  $U_{\text{UV}}(z) = 3/(4z^2)$  that yields  $e^{2A(z)} = z^{-2}$ . The conformal isometry of the metric reflects then the conformal behavior of QCD in the ultraviolet. The conformal invariance can be broken by two ways — either by introducing a hard cutoff  $z_{\text{IR}}$  [1, 2, 11] (the resulting spectrum  $m_n \sim n$  does not agree with the phenomenology in this case) or by introducing a nontrivial  $U_{\text{IR}}(z)$ . In order to obtain the desired behavior  $m_n^2 \sim n$  one has to have  $U_{\text{IR}}(z) \sim z^2$  at least at  $z \rightarrow \infty$ , *i.e.* a potential of the linear oscillator type. This idea was realized in the soft-wall models [3] by means of introduction of a dilaton field in action (1).

We want to have a spectrum of oscillator type at small  $n$  and a finite number of discrete energy levels. This is known to require a certain anharmonicity at large  $z$ . The general form of the spectrum given by such anharmonic potentials can be derived in a model-independent way by analyzing the anharmonic corrections,

$$U_{\text{IR}}(z) = \omega^2 z^2 + \alpha z^3 + \beta z^4, \quad (8)$$

where we have taken into account the  $\mathcal{O}(z^4)$  term because the first anharmonic correction arising from the  $\mathcal{O}(z^3)$  term disappears [16]. For the time being we neglect the UV contribution  $U_{\text{UV}}(z)$  (to be discussed below).

Considering the anharmonic corrections as perturbations, Eq. (5) leads to the spectrum [16]

$$m_n^2 = 2\omega(n + 1/2) - \gamma(n + 1/2)^2 + \text{const}, \quad n = 0, 1, 2, \dots \quad (9)$$

where

$$\gamma = \frac{3}{2\omega^2} \left( \frac{5}{2\omega^2} \alpha^2 - \beta \right). \quad (10)$$

It can be seen straightforwardly that one has the spectrum of oscillator type at small  $n$  if  $|\gamma| \ll \omega$  and a finite number of energy levels if  $\gamma > 0$ . The approximate independence of meson masses on  $N_c$  imposes  $\omega = \mathcal{O}(N_c^0)$ . In order to have the number of states proportional to  $N_c$  we must have  $\gamma = \mathcal{O}(1/N_c)$ , *i.e.*, the nonlinear corrections to the spectrum are subleading in the large- $N_c$  counting.

Consider now the impact of UV region introducing  $U_{UV}(z) = \varepsilon/z^2$ ,  $\varepsilon \rightarrow 0$ . The boundary condition for the oscillator wave functions,  $\psi(\infty) = 0$ , is then supplied by  $\psi(\delta) = 0$ ,  $\delta \rightarrow 0$ . As a result the even levels in spectrum (9) disappear, one has  $n = 1, 3, \dots$ . Replacing  $n$  by

$$k = n/2 - 1/2, \quad k = 0, 1, 2, \dots, \quad (11)$$

the leading in  $N_c$  contribution is given by

$$m_k^2 \simeq 4\omega(k + 3/4) + \text{const.} \quad (12)$$

Enlarging  $\varepsilon$  leads to a shift up of energy levels (see, *e.g.*, the exact example for  $\varepsilon = 3/4$  in [3]), finally some of the highest levels can vanish. All this, however, does not change the general conclusions about the qualitative behavior of the spectrum.

We assume that the general form of potentials should be such that there were no possibility for quantum tunneling to the deep infrared,  $z \rightarrow \infty$ , which we do not know how to interpret physically. Together with the requirement of finite number of discrete energy levels this leads to the following general form of our potentials: They look like a potential well (with the right wall of the form  $z^2$ ) at relatively small  $z$  and gradually transform into a plateau at large  $z$ ,  $U_{IR}(z) \xrightarrow{z \rightarrow \infty} \Lambda_{UV}^2$ , see Fig. 1. The asymptotic behavior of the metric follows from Eq. (6):

$$A(z) \xrightarrow{z \rightarrow \infty} \pm 2\Lambda_{UV} z \quad (13)$$

(see Eq. (2)). In order to have the finite action we have to choose the minus sign. The number of discrete energy levels is determined by the value of  $\Lambda_{UV}$  above which (plus an arbitrary constant) one has a continuum and by the "width" of the potential which we associate with the IR-cutoff  $\Lambda_{IR}$ . As was said above, enlarging the factor in front of  $U_{UV}(z)$  one shifts up the energy levels, as a result some of the highly excited states can disappear. The quantity  $\Lambda_{IR}$  determines (up to a presumably small constant) an effective IR-cutoff  $z_{IR}$  in the coordinate space: In the vicinity of  $z_{IR}$  the potential well ceases to be of the form  $z^2$  starting to transform into a plateau.

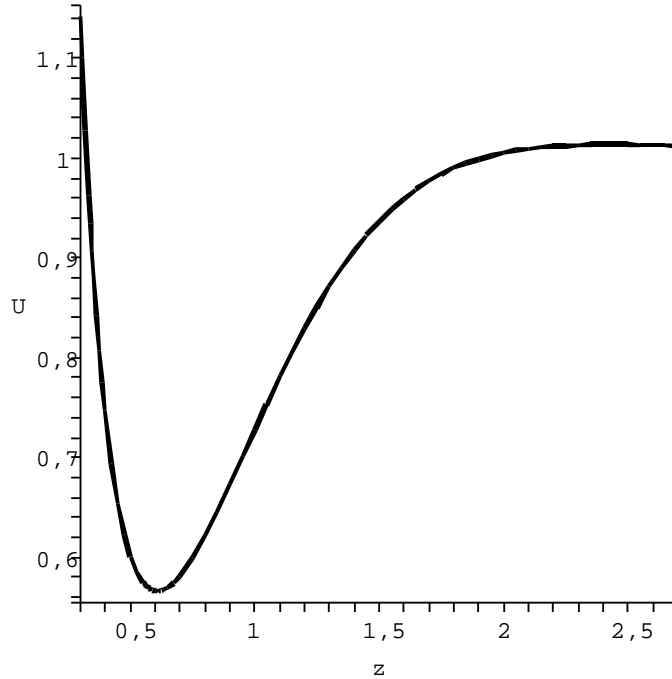


Figure 1: The general form of holographic potentials discussed in the present paper. The units are conditional.

## 4 Comparison with the experiment

Let us compare our suggestions with the experiment. In the Section "Light Unflavored Mesons", the Particle Data [9] reports five  $\rho$ -mesons:  $\rho(770)$ ,  $\rho(1450)$ ,  $\rho(1700)$ ,  $\rho(1900)$ , and  $\rho(2150)$ , the last two states are omitted from the summary table but we include them into our fit as they have been independently seen by several collaborations. Taking the experimental masses of these resonances with reported errors we can make the least square fit with the curve

$$m_n^2 = An^2 + Bn + C \quad (14)$$

and estimate the errors in values of parameters, the result is (in  $\text{GeV}^2$ )

$$m_n^2 \approx (-0.09 \pm 0.02)n^2 + (1.30 \pm 0.08)n + (0.71 \pm 0.02). \quad (15)$$

In the Section "Other Light Unflavored Mesons", the Particle Data [9] reports another two  $\rho$ -mesons:  $\rho(2000)$  and  $\rho(2270)$ . The former was seen by one collaboration only, for this reason we do not consider it. The latter was independently seen by two collaborations, its mass is in a good agreement with the fit (15).

Since the lowest three resonances are particularly well established, it may be reasonable to display also the fit based on these three states only,

$$m_n^2 \approx (-0.37 \pm 0.10)n^2 + (1.91 \pm 0.18)n + 0.06. \quad (16)$$

Thus the nonlinear correction to the spectrum is in a qualitative agreement with our analysis both in sign and in magnitude. Having fit (15) one can obtain the numerical values for the parameters of anharmonic potentials.

## 5 A toy-model

To estimate the typical values of input parameters let us consider the following example of potential having the shape "potential well + plateau",

$$U(z) = \Lambda_{\text{UV}}^2 (1 - e^{-\Lambda_{\text{IR}}(z-z_0)})^2 + C, \quad (17)$$

where  $C$  is a constant. This is the Morse potential [17] that has been widely used for description of vibrations of nuclei (near  $z_0$ ) in a diatomic molecule. In our case, by shifting  $z_0 > 0$  we can imitate  $U_{\text{UV}}(z)$  in the region of applicability of our models, thus the potential (17) is a good interpolation for the class of potentials described above. This potential has an advantage of being exactly solvable, the corresponding spectrum is [17]

$$m_n^2 = 2\Lambda_{\text{UV}}\Lambda_{\text{IR}}(n + 1/2) - \Lambda_{\text{IR}}^2(n + 1/2)^2 + C, \quad (18)$$

$$0 \leq n \leq \frac{\Lambda_{\text{UV}}}{\Lambda_{\text{IR}}} - \frac{1}{2}. \quad (19)$$

Exploring fit (15) we obtain the values of parameters<sup>2</sup>:  $\Lambda_{\text{UV}} \approx 2.34$  GeV,  $\Lambda_{\text{IR}} \approx 0.30$  GeV,  $C \approx 0.04$  GeV<sup>2</sup>. One has then seven resonances with the masses (in GeV)

$$m_n = \{0.84, 1.39, 1.72, 1.96, 2.12, 2.24, 2.31\}. \quad (20)$$

According to our previous discussions, the scaling of cutoffs in  $N_c$  is  $\Lambda_{\text{UV}} = \mathcal{O}(\sqrt{N_c})$ ,  $\Lambda_{\text{IR}} = \mathcal{O}(1/\sqrt{N_c})$ . It is interesting to note that to the leading order in  $N_c$  the mass of the ground state is (neglecting a small  $C$ ) the geometric mean of two cutoffs.

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<sup>2</sup>As seen the numerical value of  $C$  is small and compatible with  $C = 0$  within the experimental errors. For instance, as long as the mass of  $\rho(1900)$  is not well established, the value  $C = 0$  is achieved at  $m_{\rho(1900)} \approx 1.93$  GeV, with the values of cutoffs being affected slightly:  $\Lambda_{\text{UV}} \approx 2.32$  GeV,  $\Lambda_{\text{IR}} \approx 0.31$  GeV.



Potential (17) can be made a representative from our class of potentials if we set  $z_0 = 0$  and add  $U_{UV}(z) = \varepsilon/z^2$  but the model cannot be then solved analytically. At small enough  $\varepsilon$  and assuming  $\Lambda_{UV} \gg \Lambda_{IR}$  we obtain in this case that the discrete levels reside in the potential well at  $z_{UV} < z < z_{IR}$ , where  $z_{UV} \simeq \sqrt{\varepsilon}/\Lambda_{UV}$ ,  $z_{IR} \simeq 1/\Lambda_{IR}$ . At the choice  $\varepsilon = 3/4$ , the wave functions  $v_n(z)$ , where  $n$  is small enough (*i.e.* one has nearly linear spectrum) approximately coincide with those found in [3].

## 6 Discussions

The proposed class of AdS/QCD models shares the attractive features of both hard- and soft-wall holographic models: Like in the hard-wall models, there is an (effective) IR-cutoff related to confinement and there is no artificial dilaton the physical origin of which is unclear, on the other hand, similarly to the soft-wall models, one has approximately linear spectrum and the absence of ambiguity in the choice of IR boundary conditions. However, if we include the higher-spin fields in the way proposed in [3], the dilaton field seems to be inescapable<sup>3</sup> if one wants to have the slope of trajectories independent of spin  $S$  and the relation  $m_{n,S}^2 \sim n + S$  that fits well the known experimental data [18, 19]. The shape of the dilaton will be different in our case, up to a factor in the exponent it will look like  $e^{-z}$  instead of  $e^{-z^2}$  obtained in [3].

It should be noted that the coupling constant  $g_5$  entering action (1) can be obtained by matching to the high-energy asymptotics of QCD two-point correlators in the same way as in [2] (in the case of finite number of resonances, the finite energy sum rules [20] should be used). We believe, however, that such a matching is performed out the region of applicability of the models under consideration. In any case, it is not needed for the derivation of the mass spectrum we are concerned.

An important result of the considered models is that the spectrum condenses at high energies, *i.e.* the radial Regge trajectories bend down and the discrete spectrum ends near the point of zero slope. Within the flux-tube models of mesons, this effect is usually interpreted as the string breaking, hence, the proposed models are able to describe this effect: Up to model-dependent constants, the string tension in our scheme is (see Eq. (18))

$$\sigma \sim \Lambda_{UV}\Lambda_{IR}(1 - tn - t) = \mathcal{O}(N_c^0), \quad (21)$$

where

$$t = \Lambda_{IR}/(2\Lambda_{UV}) = \mathcal{O}(1/N_c), \quad (22)$$

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<sup>3</sup>Assuming that there are not additional fields in the action (1) which are dual to certain QCD operators.

*i.e.*  $\sigma$  is a constant in the large- $N_c$  limit only.

The issue of non-linear corrections to the string-like spectrum was discussed some time ago in the context of QCD sum rules in the planar limit (see, *e.g.*, [14, 21, 22]), in particular such corrections were advocated to be exponentially small,  $e^{-bn}$  with  $b > 0$ , in Ref. [21]. If one assumes the scaling  $b = \mathcal{O}(1/\sqrt{N_c})$  and expands the exponential regarding  $b$  as a small parameter, the form of exponential correction will be compatible with our results.

In the minimal version, the considered class of models has 3 parameters: The factors in front of the harmonic and anharmonic terms in the holographic potential (we can set  $\alpha = 0$  or  $\beta = 0$  in Eq. (8) without change of ensuing conclusions). This is not very constraining as long as experimentally only a few of states are known in each channel. We note, however, that the first two parameters are expected to be universal for all channels, hence, adding new channels in the model one introduces only one new parameter for each tower of resonances — a new arbitrary constant — or even less due to some degeneracies [18, 19], thus such models will be much more predictive if many kinds of mesons are described simultaneously.

Recently the necessity of anharmonic corrections to the holographic potentials was emphasized in Ref. [15] because they reflect holographically the contribution of the QCD operators from the Operator Product Expansion for the correlators of quark currents [23]. It is interesting to make the following observation: The study of the toy-model in Sect. 5 suggests an intriguing possibility that the infinite series of anharmonic corrections found in [15] might be summed up into some simple function on the 5d side.

The introduction of UV-cut-off  $\Lambda_{UV}$  may solve the following problem in building the AdS/QCD models: The 5d action is assumed to be local, *i.e.* the higher-derivative terms are supposed to be suppressed. However, there is no understanding what scale should suppress those terms. Within the AdS/QCD models put forward in the present work, one can speculate that the suppression is provided by powers of  $1/\Lambda_{UV}$ .

From the physical sense and numerical estimates for the IR-cut-off  $\Lambda_{IR}$  it is tempting to associate this quantity with  $\Lambda_{QCD}$  as in the hard-wall models [1]. According to a usual belief, however,  $\Lambda_{QCD}$  is nearly independent of  $N_c$  and this provides an approximate  $m_n = \mathcal{O}(N_c^0)$  scaling for meson masses. In our case, the situation is more tricky: Although  $\Lambda_{IR} = \mathcal{O}(1/\sqrt{N_c})$ , the scaling  $m_n^2 \sim \Lambda_{IR}\Lambda_{UV} = \mathcal{O}(N_c^0)$  holds due to the existence of the UV-cut-off  $\Lambda_{UV} = \mathcal{O}(\sqrt{N_c})$ , thus we should assume

$$\Lambda_{QCD}^2 \sim \Lambda_{IR}\Lambda_{UV}. \quad (23)$$

If the considered models indeed reflect the real QCD, for the first time we are dealing with a situation when we can learn something qualitatively new

about QCD from the AdS/QCD models. Specifically, our previous analysis suggests the following mechanism for generating  $\Lambda_{\text{QCD}}$  in the probable analytical solution for QCD: One introduces the IR and UV cutoffs and takes the limit  $\Lambda_{\text{IR}} \rightarrow 0$ ,  $\Lambda_{\text{UV}} \rightarrow \infty$  such that the product  $\Lambda_{\text{IR}}\Lambda_{\text{UV}}$  remains finite and should be associated with  $\Lambda_{\text{QCD}}^2$  that determines the mass scale of the theory with massless quarks. Dealing with an effective theory of QCD one works with finite  $\Lambda_{\text{UV}}$  and nonzero  $\Lambda_{\text{IR}}$  that are input parameters of the effective theory. Precisely this ideology lies behind the models considered in the given work.

## 7 Conclusions

In the present paper, we have designed a class of AdS/QCD models that describes a finite number of meson resonances whose spectrum is approximately Regge-like. Assuming that the number of distinguishable resonances depends linearly on  $N_c$  we obtained that the non-linear corrections to the Regge-like spectrum are subleading in  $1/N_c$ . Within the given models, the non-linear corrections stem from the anharmonic contributions to the holographic potential that determines the mass spectrum. According to the recent results [15] the anharmonic contributions on the 5d side appear after taking into account the QCD operators responsible for the masses of hadrons, therefore the design of the considered models is somewhat justified from a more fundamental standpoint. The presented approach and that of [15] share also the following important property: The slope of radial trajectories is, up to a dimensionless factor of the order of unity, a product of IR and UV cutoffs of the model, *i.e.* the discrete mass spectrum is equally determined by both IR and UV sectors of the theory. This interesting feature suggests that something similar may happen in the exact solution for the mass spectrum of real QCD.

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